A Precis of *Minimal Foundations*

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This handout is designed to accompany a largely non-technical talk on the main ideas of my book draft, *Minimal Foundations: A Humean Account of Metaphysical Analysis*. The handout follows the order of the talk, while recording a number of the formal principles and definitions that matter in the background. In the book I develop a theory of metaphysical analysis and I articulate and defend a form of *metaphysical foundationalism*, according to which there are atoms of metaphysical analysis ("fundamental entities"), in terms of which everything has a unique metaphysical analysis. The theory implies strong supervenience principles and Humean principles of modal plenitude, and is strong enough to provide logicist treatments of modality and mathematics.

The theory is, moreover, logically precise. It is officially formulated in a higher-order language and one can reason about metaphysical questions inside the system, in much the same way that mathematicians can formulate and reason about mathematical questions in Zermelo-Fraenkle set theory.

1. Metaphysical definition

The world is rich: there are ordinary objects, social objects, mathematical truths, modal truths, ethical truths, and so on. Do these things form a metaphysically unstructured jumble, or are some things and facts more basic than others? If so, what is the theoretical role of this relation of "being more basic"? Some relations that have been employed in this connection include supervenience, and ground. The book puts *metaphysical analysis* at center stage. It starts with the thought that one thing may be *metaphysically definable* in terms of some others, in rough analogy with the way a complex expression in a language may be defined from simpler expressions.

My goal is not to uncover the structure of some preotheoretic notion of metaphysical analysis—for me it is a theoretical term defined by it's role with respect to more familiar concepts and projects. Precisifying the notion may have to involve departures from naïve impressions about how it should behave. Nonetheless, I see the book as a continuation of a longstanding quest in philosophy to understand the logical structure of reality in terms of metaphysical analysis, whose protagonists include Leibniz, Bolzano, the logical atomists, Lewis and Sider.

To take a familiar example of linguistic definition: 'is a bachelor' may be defined from 'is a man' and 'is married to'. The book's central question is how to make the metaphysical analogue precise without assuming that reality literally has the structure of a language.

> **Metaphysical definition (informal).**
>
> An entity is metaphysically less basic than some others when it can be obtained from them by a purely logical operation.

The logical operation is the "metaphysical analysis". Here are some examples .

- Bachelor $= \lambda x.(\mathrm{Man}\, x \wedge \neg\exists y.\, \mathrm{Married}\, xy)$

  The logical operation (i.e. the metaphysical analysis) is $\lambda FRx.(Fx \wedge \neg\exists y.Rxy)$

- Above $= \lambda xy.\,\text{Below}\,yx$.

  The logical operation is $\lambda Rxy.Ryx$.

- Let S be a shape property of infinite complexity. S is metaphysically definable from Hilbert-Tarski primitives of geometry, congruence and betweenness.

  In this case there is no finite definition, but we can still quantify over metaphysical analyses. I thus theorize with a predicate at each type Pure_σ that applies to the purely logical operations at that type.

The later formal theory uses a primitive definability relation, written $\rhd$, which is interdefinable with purity, to regiment this idea. I return to that in the appendix; see especially Table 1. For a fuller technical presentation, see my MS *Logics of Metaphysical Definition*.

Two disanalogies with *linguistic* definition matter.

- The constituents of expressions in human languages appear in a linear order because of contingent facts about us: we speak one word after another. It would be surprising if reality itself were structured in exactly that way.
- The Russell–Myhill family of paradoxes forces some divergence from the simple linguistic picture. On the view pursued here, one should not in general distinguish statements that are logically equivalent in a sufficiently minimal higher-order logic of connectives and quantifiers. The background logic is called "Classicism", described in *The Broadest Necessity* (2018) and explored in my paper with Cian Dorr, *Classicism* (2024).
- Another disanalogy is about *uniqueness*. A natural first thought is that the complete fundamental inventory should stand to the rest of reality as the primitive vocabulary of a language stands to its complex expressions. But the better analogy, on this view, is often with a basis for a vector space: there may be more than one complete and independent choice of basis. So the fundamental inventory need not be uniquely realized. For instance: if there is a fundamental inventory including *being more massive than*, then the result of switching that relation for *being less massive than* is also a fundamental inventory. (*Pace* Sider on truth functional bases.)

In light of the last point, I have, in addition to the purity primitive, a primitive relation FunInv at each relational type stating that the arguments form a fundamental basis or 'inventory'.

2. Metaphysical foundationalism

The book's official version of metaphysical foundationalism is intentionally spare. It is not, first of all, a thesis about which things are fundamental; it is a structural thesis about what a fundamental inventory must do.

Fundamental Completeness (informal).
Every entity has at least one metaphysical definition in terms of the fundamental entities.

$$\text{FunInv}_{\overline{\sigma}\to t}\, x_1 \dots x_n \to \forall_\tau y\, \exists_{\overline{\sigma}\to\tau} Q\big(\text{Pure}(Q) \wedge Qx_1 \dots x_n = y\big).$$

Fundamental Independence (informal).

Every entity has at most one metaphysical definition in terms of the fundamental entities.

$$\begin{aligned}&\mathrm{FunInv}_{\overline{\sigma}\to t}\, x_1 \ldots x_n \to \\ &\forall_\tau y\, \forall_{\overline{\sigma}\to\tau} P\, \forall_{\overline{\sigma}\to\tau} Q \Big(\mathrm{Pure}(P) \wedge \mathrm{Pure}(Q) \\ &\qquad \wedge\ P x_1 \ldots x_n = y \wedge Q x_1 \ldots x_n = y \to P = Q\Big).\end{aligned}$$

These principles are schematic in type. They say that the fundamental inventory is both *complete* and *independent*: everything is definable from it, and nothing receives two distinct analyses from it.

It is worth stressing what is *not* built in. The framework does not tell us that the fundamental must be spacetime points, or fields, or tropes, or sets, or anything else in particular. In the book I use spacetime physics as a placeholder because it models the kind of minimality at issue, not because the view is meant to prejudge future physics.

The other important point is that the completeness and independence conditions do not force a uniquely satisfied notion of fundamentality. Just as a vector space can have many bases, reality may admit many complete and independent fundamental inventories. That is one way in which metaphysical analysis does not perfectly reflect linguistic analysis, where the linguistic primitives of a language are unique.

3. Hume's dictum

Hume's dictum is introduced as a denial of non-trivial necessary connections between distinct things. In this book it is understood in a particularly strong and systematic way: the broadest notion of necessity should not connect distinct fundamental entities except in trivial, purely logical ways.

Hume's Dictum.

Any broadly necessary logical connection between distinct fundamental entities is trivial.

$$\mathrm{Fun}\, \overline{x} \wedge \mathrm{Pure}(Q) \to \Box Q\overline{x} \to Q = \lambda\overline{y}.\top.$$

The contrapositive form makes the intended modal content vivid.

Hume's Dictum, contraposed.

Distinct fundamental entities can be logically arranged in any consistent way.

$$\mathrm{Fun}\, \overline{x} \wedge \mathrm{Pure}(Q) \wedge Q \neq \lambda\overline{y}.\bot \to \Diamond Q\overline{x}.$$

Informally, the guiding thought is that all necessities must ultimately come from identities. If two quantities, properties, or propositions covary necessarily, then that necessary connection had better be explained by their not really being distinct after all. Otherwise we are left with unexplained "modal magic."

Some consequences emphasized in the introduction are these:

- **Parsimony.** If two candidate quantities necessarily coincide, Hume's dictum pressures us to identify them rather than posit both. E.g. velocity is not fundamental, if position at a time is; it is metaphysically analysed in terms of derivatives.

- **Intensionalism.** Necessarily equivalent propositions, properties, and relations are the same.
- **Recombination.** The fundamental is freely recombinable: if a pure arrangement is consistent, it is possible.
- **Mathematical nominalism.** Mathematical truths are better understood as truths about highly general patterns than as brute necessities about abstract objects.

This notion of "logical necessity" is therefore stronger in some ways and stranger in others than the familiar Kripkean notion of metaphysical necessity. In particular, the package is friendly to contingent distinctness and does not treat S5 as the logic of the broadest necessity.

4. Relationships between these principles

A central theme of the book is that Humean modal plenitude is not an optional add-on to metaphysical foundationalism. It is deeply tied to the structural role of a fundamental inventory.

Proposition. *Fundamental Independence implies Hume's Dictum.*

Very roughly: if a necessary pure connection Q holds of the fundamental, then $Q\overline{x}$ is a necessary proposition, hence identical to $\top$. But by Fundamental Independence, the metaphysical analysis of a proposition in terms of the fundamental is unique. So the analysis delivered by Q must coincide with the trivial analysis.

Proposition. *Hume's Dictum implies Fundamental Independence.*

So the two principles line up exactly:

> **Equivalence.**
>
> Fundamental Independence and Hume's Dictum are equivalent.

The book also formulates a higher-order supervenience thesis.

> **Fundamental Supervenience.**
>
> Every possibility is entailed by some possible arrangement of the fundamental inventory.

$$\mathrm{FunInv}\,\overline{e} \to \forall_t p\Big(\Diamond p \to \exists_{\overline{\sigma}\to t} Q\big(\mathrm{Pure}(Q) \wedge \Diamond Q\overline{e} \wedge \Box(Q\overline{e} \to p)\big)\Big).$$

Proposition. *Fundamental Completeness entails Fundamental Supervenience.*

Intuitively: if every proposition already has a metaphysical definition from the fundamental, then every possibility is trivially entailed by how the fundamental are arranged.

The converse requires some further assumptions: propositions, properties and relations have infinitary conjunctions, and that the pure operations are closed under these operations.

Proposition. *Assuming Boolean Completeness and Pure Infinite Closure, Fundamental Supervenience implies Fundamental Completeness at type t.*

I also discuss two weaker principles.

Fundamental Primeness.

No fundamental entity can be defined from the others.

$$\text{FunInv}\ \overline{x}\, y \to \neg\exists Q\big(\text{Pure}(Q) \wedge Q\overline{x} = y\big).$$

Minimal Supervenience.

The fundamental form a minimal supervenience base.

$$\text{FunInv}\,\overline{e} \to \text{MinSupBase}\,\overline{e}.$$

Proposition. *Fundamental Independence entails Minimal Supervenience.*

Proposition. *Minimal Supervenience entails Fundamental Primeness.*

Minimal Supervenience and Fundamental Primeness are strictly weaker than Hume's Dictum/Fundamental Independence, and are maybe more intuitive or familiar constraints. However they share many of the same strange implications that Hume's Dictum and Fundamental Independence have.

5. Modal Logicism

Suppose the fundamental inventory contains only spacetime points, fields, and fundamental relations among them. Where are the modal facts to be found? The actual distribution of fields may explain why David Lewis in fact died when he did, but it does not obviously explain why he *could have* died differently. The distribution of geometric relations over the spacetime points contain information about the actual geometric structure of space, but do not seem to contain information about which alternative geometries — Euclidean, Gödelian, etc – are broadly possible.

The book's answer is modal logicism.

Modal Logicism.

The central concepts in modal metaphysics — including being a necessity, comparative breadth among necessities, propositional entailment, necessity in the highest degree, and the notion of a possible world — can be defined in purely logical terms from higher-order quantification and truth-functional structure.

The definitional hierarchy goes as follows

- First define, in purely logical terms, the notion of an operator with a normal modal logic.
- A *necessity* is an operator that is *necessarily* normal, with respect to every normal notion of *necessarily.*
- Then define when one necessity is *broader* than another.
- From that define *broad necessity*, or necessity in the highest degree.
- Then define *entailment* and, finally, *possible worlds* in terms of maximal consistent propositions.

A representative definition is the one for broad necessity:

> **Broad Necessity.**
> A proposition is broadly necessary iff it is necessary according to every necessity.
>
> $$\Box := \lambda p.\forall_{t\to t}\Box_i(\mathrm{Nec}(\Box_i) \to \Box_i p).$$

And two corresponding world-notions are:

> **Weak and strong world propositions.**
>
> $$\mathrm{WWorld} := \lambda w(\Diamond w \wedge \forall_t p\,(w \leq p \vee w \leq \neg p)),$$
> $$\mathrm{SWorld} := \lambda w(\Diamond w \wedge \Box\forall_t p\,(w \leq p \vee w \leq \neg p)).$$

The broader philosophical ambition is reduction: if modality can be reconstructed in purely logical terms, then modal facts need not be read off from a special stock of irreducible modal primitives in the base ontology.

6. Mathematical Logicism

We can similarly ask where the mathematical facts are to be found in our minimal fundamental inventory, and the account is parallel. The minimalist wants to avoid taking numbers, sets, functions, and the rest as part of the fundamental furniture. But there are clearly highly general mathematical patterns in nature. The proposal is to identify mathematical truths with truths about such patterns, expressed using higher-order resources rather than a special ontology of abstract objects.

> **Mathematical Logicism**
> Mathematical facts are general patterns in nature, and can be encoded as truths about *maximally specific structural properties.*

To recover mathematics, logicists have typically needed to make special extra assumptions of mathematical plenitude: the axiom of infinity, that there is an abundance of sets, or categories or what have you. My approach is compatible with finitism about individuals, the necessary principles of plenitude of maximally specific structural properties falls out of the existing theory of metaphysical analysis.

So, for example, one can associate to an arithmetical statement quantifying over numbers a general pattern expressed by quantifying into the position of numerical quantifiers. Here is the road map.

- **Arithmetic**: count numerical quantifiers (e.g. "λF.there are N Fs"). The maximally specific structural properties of plain collections of objects are just their number.
- **Analysis**: mass numerical quantifiers (e.g. "λFG. there is α times as much F as G"). These are maximally specific structural properties of collections endowed with parthood and size.
- **Ordinals**: maximally specific structural properties of well-orders, that is, properties of having a certain order-type.

- **Sets**: maximally specific structural properties of well-founded extensional trees.

What matters is that the relevant pattern can obtain whether or not there are special abstract objects corresponding to it. If seven pebbles cannot be arranged into a non-trivial rectangle, that pattern is there in nature independently of whether the number 7 is taken to be an abstract object.

For illustration, here are the definitions of *set formation*, *membership* and *set* in this framework. Let G be a rigid property of set-like structures. Then $\{G\}$ is the property a tree u has just in case every G-set is instantiated, and the immediate subtrees of u are exactly the trees instantiating some member of G.

Set Formation.

$$\{G\} := \lambda u\Big(\mathrm{Tree}(u) \wedge \forall X(GX \rightarrow \exists x\, Xx) \wedge \forall x(x \lhd u \leftrightarrow \exists X\,(GX \wedge Xx))\Big).$$

From this the chapter defines membership:

Membership.

$$\in := \lambda XY.\exists^{Rg} G(GX \wedge Y = \{G\}).$$

Definition of Set

An entity X is a *set* iff, for every property V, whenever V is closed under consistent set formation—that is, whenever for every rigid consistent property G, if every instance of G is a V-entity, then the entity $\{x \mid Gx\}$ is also a V-entity—it follows that X is a V-entity.

Formalization.

$$\mathrm{Set} := \lambda X.\forall V\Big(\Big[\forall G\Big((\mathrm{Rigid}(G)\wedge\mathrm{Consistent}(G)\wedge\forall y(Gy \rightarrow Vy)) \rightarrow V(\{x \mid Gx\})\Big)\Big] \rightarrow VX\Big).$$

The important this is not merely that these definitions can be written down, but that the required modal plenitude falls out of the broader foundational picture: Humean recombination, together with the claim that structural properties are logical and therefore available "for free" in metaphysical analysis, supplies the modal room needed for the logicist constructions.

7. Appendix: The logic of metaphysical definition

The primitive of the formal theory is a relation of metaphysical definability, written $\rhd$. Very roughly, a sequent

$$\Gamma \rhd_{\overline{x}} A$$

where Γ stands for a sequence of terms, $B_1, \ldots, B_n$, this statement says that the items in Γ metaphysically define A, or, there is a metaphysical analysis of A in terms of $B_1, \ldots, B_n$. If the parameters $\overline{x} = x_1 \ldots x_k$ appear free in $A, B_, 1 \ldots, B_n$ then the definition has to be uniform in arbitrary values of the parameters. More carefully, $\rhd$ is a family of relations for each possible types for the definiens and definiendum.

We can also define it in terms of our notion of a pure operation (a metaphysical analysis)

Metaphysical definition and metaphysical analysis

$\Gamma \rhd_{\overline{x}} A$ means there is a metaphysical analysis of A in terms of Γ, that is uniform in the variables $\overline{x}$

$$B_1, \ldots, B_n \rhd_{\overline{x}} A := \exists Q(\mathrm{Pure}\, Q \wedge \lambda\overline{x}.PB_1 \ldots B_n = \lambda\overline{x}.A)$$

Conversely we can define the notion of a metaphysical analysis from the notion of metaphysical definition as something definable from nothing.

$$\mathrm{Pure} := \lambda X. \rhd X$$

The notation $\overline{x}$ abbreviates a finite sequence $x_1, \ldots, x_n$. Thus $\mathrm{Fun}\, \overline{x}$ says that $x_1, \ldots, x_n$ are distinct and fundamental, while $Q\overline{x}$ abbreviates the application $Qx_1 \ldots x_n$.

The table below reproduces the core proof-theoretic principles from the draft. It is this package that underwrites the talk's informal use of *metaphysical definition*.

Table 1: Axioms of metaphysical definition

Principle	**Schema**
Instantiation	$\Gamma \rhd^{\overline{\sigma}\rho}_{\overline{x}y} A \to \Gamma[B/y] \rhd^{\overline{\sigma}\rho}_{\overline{x}z} A[B/y]$
Cut	$(\Gamma \rhd^{\overline{\theta}\sigma}_{\overline{x}} A) \wedge (\Delta, A, \Delta' \rhd^{\overline{\rho}_1\sigma\overline{\rho}_2}_{\overline{y}} B) \to (\Delta, \Gamma, \Delta' \rhd^{\overline{\rho}_1\theta\rho_2}_{\overline{y}} B)$
Concretion	$\Gamma \rhd^{\overline{\rho}(\sigma\to\tau)}_{\overline{x}} F \to \Gamma, y \rhd^{\overline{\rho}\sigma\tau}_{\overline{x}y} Fy$
Id	$x \rhd^{\sigma}_{y} x$
Switch	$\Gamma, x, y, \Delta \rhd_{\overline{z}} C \to \Gamma, y, x, \Delta \rhd_{\overline{z}} C$
Merge	$\Gamma, y, \Delta, y, \Sigma \rhd_{\overline{x}} C \to \Gamma, y, \Delta, \Sigma \rhd_{\overline{x}} C$
Vac	$\Gamma \rhd_{\overline{x}} C \to \Gamma, y \rhd_{\overline{x}y} C$
Abs	$\Gamma, y \rhd^{\overline{\rho}\sigma\tau}_{\overline{x}y} My \to \Gamma \rhd^{\overline{\rho}(\sigma\to\tau)}_{\overline{x}} M$
Decomp	$\Gamma, A \rhd^{\overline{\rho}\sigma}_{\overline{x}} B \to \exists_{\overline{\rho}\to\sigma} F\big(\Gamma \rhd^{\overline{\rho}\to\sigma}_{\emptyset} F \wedge \lambda\overline{x}.FA =_\sigma \lambda\overline{x}.B\big)$

Further reading

- Andrew Bacon, *Minimal Foundations: A Humean Account of Metaphysical Analysis*, draft dated December 31, 2025.
- Andrew Bacon, *The Broadest Necessity.*
- Andrew Bacon and Cian Dorr, *Classicism.*
- Andrew Bacon, *Logics of Metaphysical Definition* (MS).