In Defence of a Naïve Conditional Epistemology

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1 The data

You pick a card at random from a standard deck of cards. How confident should I be about asserting the following sentences?

1. The selected card is an ace if it's red?
2. It's spades if it's black?
3. It's diamonds if it's an eight?

To be clear: I mean how confident should I be in the propositions I would assert by uttering these sentences.

The obvious answer to these questions are: $\frac{1}{13}$, $\frac{1}{2}$, $\frac{1}{4}$, ...

Note: triviality results don't obviously refute particular judgements like these. How do we explain this systematic connection?

2 Stalnaker's Thesis

An initially attractive thesis, that explains the relevant data.

> Stalnaker's Thesis: The degree of belief one should assign to a conditional sentence, 'if A then B', should be identical to ones conditional degree of belief in B given A.[1]

The thesis, as formulated above (and by Stalnaker), is not careful enough about use and mention. In what follows I shall read the thesis as saying that, if p, q and r are the propositions that would be asserted by the sentences A, B and 'if A then B' in a given context, then ones degree of belief in r must be identical to ones conditional degree of belief in q given p.

2.1 Stalnaker's thesis and contextualism

Enthusiasm for this theory (amongst truth conditional theorists) was short lived due to the triviality results. Another, seemingly independent reason to worry about it if you are a contextualist.

[1] If Cr is a function representing your degrees of belief, then your conditional degree of belief in B given A, $Cr(B \mid A)$, is defined to be $\frac{Cr(A \wedge B)}{Cr(A)}$ when $Cr(A) > 0$.

CONTEXTUALISM: (i) conditional sentences can be used to assert propositions, (ii) which proposition is asserted by a conditional sentence can depend on the context of utterance even when the antecent and consequent do not and (iii) which proposition is asserted depends on some contextually salient piece of evidence or knowledge.

Of course there is a rich philosophical tradition that rejects (i). Nonetheless, if you are a truth conditional theorist there are a number of puzzles that contextualism helps explain. Among them:

> **Or-to-if inferences**: the apparent validity of the move from 'either not A or B' to 'if A then B', which leads to the collapse of the indicative conditional to a material conditional.
>
> **Gibbard cases**: cases where two people are apparently able to truthfully assert 'if A then B' and 'if A then not B' (when A is not known to be false by any participants of the conversation.)

Stalnaker's thesis, as precisified above, makes little sense if conditionals are context sensitive. The problem, in essence, is that there are many different propositions a conditional sentence can express, and only one conditional probability for the probability of those conditionals to be identical to. The point is that even when A and B are not context sensitive, 'if A then B' can express many different propositions with potentially different probabilities, whereas the conditional probability is determined determined only by the propositions expressed by A and B.

2.2 Lewis's result

If $\mathcal{C}$ is a class of probability functions that is closed under conditioning then either there are functions in $\mathcal{C}$ which do not satisfy Stalnaker's thesis or the probability of any conditional with non-zero probability antecedent is the probability of it's consequent.

> Proof: let $\rightarrow$ be the connective expressed by an indicative conditional in some context and suppose that Stalnaker's thesis does hold throughout $\mathcal{C}$. Then for any A and B and $Cr \in \mathcal{C}$, $Cr(A \rightarrow B) = Cr(A \rightarrow B \mid B)Cr(B) + Cr(A \rightarrow B \mid \neg B)Cr(\neg B)$ by probability theory. Applying Stalnaker's thesis it follows that this $= Cr(B \mid AB)Cr(B) + Cr(B \mid A\neg B)Cr(\neg B) = 1.Cr(B) + 0.Cr(\neg B) = Cr(B)$

3 Revising the theory

The conclusion: Stalnaker's thesis (or at least the above precisification of it) is false. *But* it wasn't particularly plausible from a contextualist perspective in the first place.

What to make of these problems? One radical response, often made in connection to the triviality results, is to take these highly theoretical arguments to undermine the original probability judgments to 1-3. To my mind this response is excessive: triviality results do not cast doubt on particular probability judgments such as those reported in 1-3. They merely refute a general theory that predicts those judgments – the judgments themselves do not imply the refuted theory. Furthermore, if the answers I listed to 1-3 are not correct then those who make the radical response owe us an answer to the question: what are the correct answers to these particular questions? If they are not respectively $\frac{1}{13}$, $\frac{1}{2}$ and $\frac{1}{4}$ then what on earth are they?

What we want is a *better* theory. One that (a) predicts the probability judgments (b) is at least compatible with contextualism about conditional statements and finally (c) is provably consistent (over a rich set of probability functions, with a plausible logic, et cetera.) In my view the contextualist has a very natural theory that satisfies all these constraints.

Note that in order to determine the probability of an utterance of a conditional we need to know the following two things:

1. In order to know what proposition is expressed by the utterance we need to know what the contextually salient evidence, E, is. What proposition is expressed depends on the evidence.

2. In order to know what the probability of that proposition is (i.e. once we've determined E) we need to know how probable it is which depends on (a) your priors, Pr, and crucially (b) your total evidence E'.

Thus two pieces of evidence play a role in this theory. The first piece of evidence determines the proposition to evaluate, the second determines how probable that proposition is.

> Hypothesis: in the context described in the examples 1-3 above the two pieces of evidence (the contextually salient evidence, and agents evidence) are identical.

The principle that would predict this: when the contextually salient evidence is E, and the agents total evidence is also E, the probability of the proposition expressed by 'If A then B' should be the probability of the proposition expressed by B conditional on the proposition expressed by A.

The proposition expressed: to represent the proposition expressed by a conditional sentence in a context where E is salient I shall use the notation $A \rightarrow_E B$. In the null context, where E is tautologous, I shall simply write $A \rightarrow B$, I'll call this the ur-conditional. (Note: a substantive principle states that $A \rightarrow_E B := A \wedge E \rightarrow B$. This is true in one of my models, but I will not assume this in what follows.)

The probability function: I shall assume *Bayesianism*: your current probability gets determined your total evidence E and your ur-priors (credences you'd have if you had no evidence at all) by conditioning the latter on the former. Thus your current probability is just $Pr(\cdot \mid E)$.

A few consequences of our notation

- Your current *conditional* credence in B given A is just $Pr(B \mid A \wedge E)$.
- Your current credence in $A \rightarrow_E B$ is just $Pr(A \rightarrow_E B \mid E)$.

Thus we can state our new principle as follows:

> The New Thesis: For any ur-prior Pr and possible evidence E, $Pr(A \rightarrow_E B \mid E) = Pr(B \mid A \wedge E\})$

Good things about CP (The New Thesis):

- It predicts the data.

- It handles the context sensitivity.
- It is consistent (it has a possible worlds semantics.)

Notes about the tenability result:

- There are actually two distinct models.
- They are both based on a selection function based semantics, a slight variant of Stalnaker's semantics (see the logic below.)
- They provide a uniform interpretation of the conditional (other tenability results can be modified to give something *a bit* like the new thesis, at the cost of making the structure of the space of worlds depend on the context!)
- There is no restriction on the way that conditionals iterate! This distinguishes the result from many other similar results.
- However in one of the models your total evidence cannot be hypothetical (although your evidence can properly include hypothetical propositions.) In this model the ur-selection function satisfies 'Harper's condition': $f_E(A, x) = f(A \cap E, x)$.
- In the other model arbitrary propositions can be your total evidence, although it does not satisfy Harper's condition.

3.1 How does it avoid Lewis's result?

Technically speaking there are lots of different conditionals, $A \rightarrow_E B$, for each evidence E.

- The ur-conditional, $A \rightarrow B$, avoids it because the ur-conditional only satisfies Stalnaker's thesis for ur-priors, and these are not closed under conditioning.
- The conditionals $A \rightarrow_E B$ only satisfy the thesis for probability functions of the form $Pr(\cdot \mid E)$, and these are not closed under conditioning.
- Consequence: the probability of $A \rightarrow_E B$ and the conditional probability of B on A can come apart when acquire new evidence.
- However: our intuitive probability judgments don't seem to track anything this strong. When you have required new evidence, E', we only need the conditional probability connection with $A \rightarrow_{E'} B$.

4 Static Triviality Results

Lewis's result is a dynamic triviality result. It shows that probability conditional-conditional probability link cannot be preserved throughout a class of probability functions. A static triviality result shows that the thesis cannot hold for even a single probability function. Note that the new thesis requires the connection to hold for each conditional.

Hájek and Hall, building on an argument by Stalnaker, show that any conditional satisfying:

CSO $A \leftrightarrow B, A \to C \vdash B \to C$.

cannot support CP. (In terms of selection functions this just means that if $f(A,x) \in B$ and $f(B,x) \in A$ then $f(A,x) = f(B,x)$.)

CSO entails limited forms of antecedent strengthening and transitivity

CM $A \to B, A \to C \vdash AB \to C$

RT $A \to B, AB \to C \vdash A \to C$

Conclusion: we must give up these principles. Here are three sources of unease people have about this.

1. Direct intuitions about the validity of CSO (and related principles.)

2. The principle is validated in semantics based on the idea of closeness.

4.1 Direct intuitions

Firstly: it's a complicated principle – no-one really has good intuitions about it.

Secondly, direct intuitions overgenerate:

> CSO might seem valid because it is a weakening of the transitivity of the conditional $A \to B, B \to C \vdash A \to C$.
>
> Transitivity, however, is highly controversial: it has several putative counterexamples and it straightforwardly entails antecedent strengthening.

Thirdly, there is actually a fairly non-trivial literature giving counterexamples to CSO. (Tichy, Maartenson, Tooley, Ahmed.)

Finally, there are many to explain away the validity of CSO.

1. In the model I provide the above principles are valid when A, B and C are restricted to categorical propositions. All the failures (at least in this model) involve iterated conditionals.

2. The principle CSO (and the others mentioned above) when formulated using the connective $A \to_E B$ are all probabilistically valid in the sense that they get probability one for every probability function of the form $Pr(\cdot \mid E)$. That is, any utterance of CSO will express a proposition you are fully confident in. For example, CSO for the ur-conditional gets probability one relative to all ur-priors (and thus, also, relative to all rational credences.)

4.2 Closeness based semantics

This is, in my view, the strongest case for CSO. Two questions:

> Should closeness play an role in the semantics of indicative conditionals. (We can remain neutral on the case of subjunctives, although my view is 'no' in both cases.)
>
> Can we still model indicatives using selection functions, and if so, how should we interpret the functions?

An apparent conflict:

> Unlike in the case of subjunctives, conditional excluded middle seems to be uncontroversially true for simple past tense indicatives. See: 'either the coin landed heads if it was flipped or it landed tails'.
>
> On a closeness interpretation of the selection function the only way to validate CEM is if we stipulate that there is *always a unique world that is closest to the actual world*. This is well known to be controversial.
>
> Lewis's semantics for counterfactuals allows there to be multiple equally close worlds. However it comes at the cost of invalidating CEM. This may be not obviously problematic for counterfactuals, where CEM is more controversial, it is not suitable for simple past tense indicatives.

Edgington's counterexample. Car has some unknown amount of petrol in it, which will make it run out somewhere between 0 and 100 miles. In fact it runs out at 36 miles. By closeness assumption:

> If the car ran for at least 50 miles it ran for exactly 50 miles.

How to understand the selection function without assuming a closeness based semantics?

> For counterfactuals: $f(A, x)$ is the world that would have obtained (instead of x) had A obtained.
>
> For indicatives: $f(A, x)$ is the world that obtains if A obtains.

Stalnaker makes the substantive assumption that the world that would have obtained if A had obtained is just the closest world. Another story might say: if there is more than one closest world it might be randomly selected from among them (compare Schulz [REF].) This would have the effect of validating CEM without requiring uniqueness.

4.3 Standardness

Finally there's the feeling that CSO is somehow part of a standard logic of conditionals and a principle of epistemic conservativeness should mean that we should have good reasons to revise it. I don't have much to say about this except that my view is that we do have good enough reasons (but I'm also inclined to deny that it's a standard logic for indicatives in the first place.)

The logic you have instead is just the following:

- **RCN** if $\vdash \psi$ then $\vdash \phi \to \psi$
- **RCEA** if $\vdash \phi \equiv \psi$ then $\vdash (\phi \to \chi) \equiv (\psi \to \chi)$
- **CK** $(\phi \to (\psi \supset \chi)) \supset ((\phi \to \psi) \supset (\phi \to \chi))$
- **ID** $\phi \to \phi$
- **MP** $(\phi \to \psi) \supset (\phi \supset \psi)$
- **C1** $(\phi \to \psi) \supset ((\psi \to \bot) \supset (\phi \to \bot))$
- **CEM** $(\phi \to \psi) \vee (\phi \to \neg\psi)$